An Empirical Study on Optimal Reinsurance about crop insurance in China

——Based on data from Inner Mongolia, Jilin and Liaoning

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Outline

- Motivation
- Model and data
- Results
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Motivation

- Agricultural Insurance
  - Adverse weather, flood, draught, hail etc.
  - Diseases of livestock.
  - Resembles catastrophe insurance.
  - Reinsurance is a major risk transfer instrument.
- Agricultural Insurance in China
  - The world's second-largest agricultural insurance market.
  - Crop insurance accounts for over 90%.
  - Government subsidy since 2007.

Motivation

- Reinsurance of crop insurance during 2008-2012
  - Cumulated original premiums: 62.68 billion RMB
  - Cumulated ceded premiums: 9.50 billion RMB.
  - Cumulated losses recovered from reinsurers: 4.57 billion RMB.
  - Cumulated expenses recovered from reinsurers :2.33 billion RMB.
  - Cumulated net ceded profit:2.13 billion RMB.
  - Ceded profit accounts for 3.4% of original premiums.
  - Ceded profit accounts for 22.42% of ceded premiums.

Motivation

- Considering the high cost of reinsurance for crop insurance:
  - How to determine the optimal reinsurance arrangement?
  - And what is the appropriate cost for it?

Model and data

- Empirical model: $\begin{cases} \min_f \rho\widehat{(x,f)} \\ s.t.\ 0 \le f_i \le x_i \\ \widehat{\pi(f)} \le \pi \end{cases}$
  - x =(x_1, x_2,…, x_n), n-dimension samples, can be collected directly, or generated randomly.
  - for loss x_i , the insurer cedes f_i to a reinsurer.
- Empirical models can be transformed into Second Order Conic Programming problems(Weng, 2009)
- CVX MATLAB toolbox (Grant et al., 2013) can be used to solve this problem

Model and data

- Advantages of empirical model (Weng,2009)
  - Simple, intuitive, practical.
  - It exploits directly the observed data.
  - Applies to a number of premium principles.
- Premium principles
  - Expectation principle: safety loading is unrelated to the variation of the risk:

$$\pi(f) = (1+\beta)E[f(X)]$$

  - Standard deviation principle: safety loading is positively related to the variation of the risk:

$$\pi(f) = E[f(X)] + \beta\sqrt{Var[f(X)]}$$

Model and data

- Risk measures
  - $VaR_X(\alpha) = inf\{x: \Pr(X > x) \leq \alpha\}$
  - $\mathrm{CTE}_X(\alpha) = E[X|X > VaR_X(\alpha)]$
  - CTE is a better risk measure compared to VaR (Cai et al., 2008).
- In this study
  - To minimize the risk measure under the reinsurance premium budget constraint.

Model and data

- Other assumptions
  - Risk tolerance level α: 2%, 5%, 10%, 20%.
  - Reinsurance premiums budget π:

  3%, **3.75%**, 4%, 5% of total premiums.
  - Safety loading coefficient β: 20%, 30%, 40%, 50%.
  - Samples size: 1000.

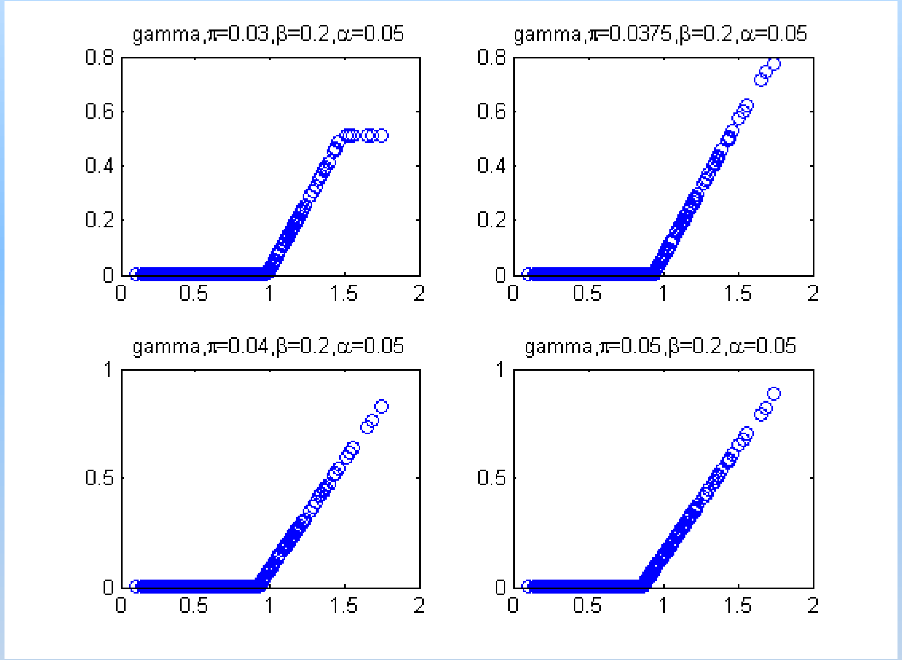
Model and data

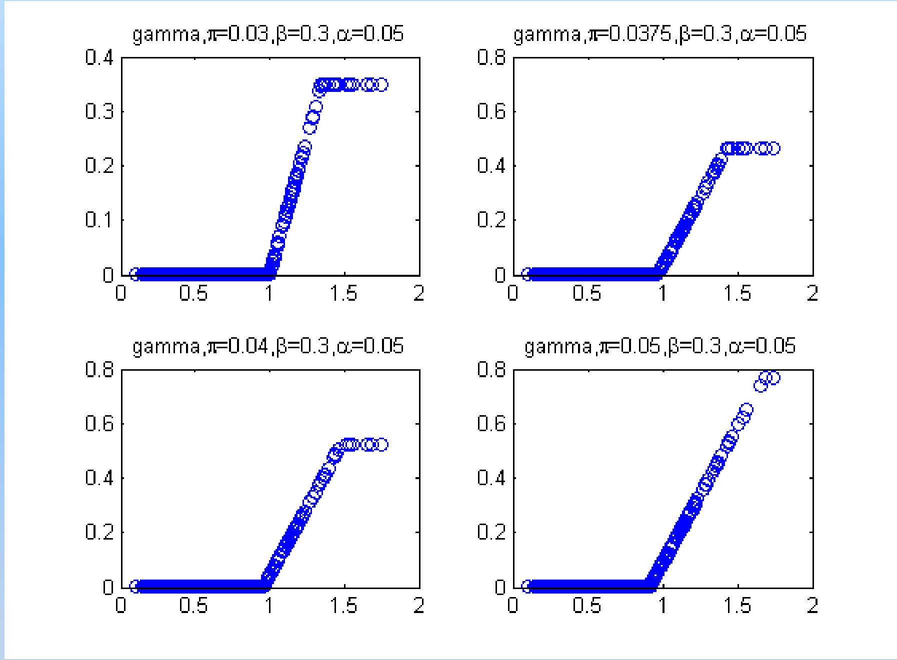
- Loss ratio distribution(AHCRES)
  - Inner Mongolia: Gamma Distribution, a = 5.74987 and b = 0.105108.
  - Liaoning: Gamma Distribution, a = 4.1405 and b = 0.1796.
  - Jilin: Generalized Extreme Value Distribution, a = 0.105086, b = 0.173391 and c = 0.53431.
  - Gamma Distribution: $f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{\frac{-x}{b}}$
  - Generalized Extreme Value Distribution:

$$f(x) = (\frac{1}{b}) exp\left(-\left(1 + a\frac{(x-c)}{b}\right)^{-\frac{1}{a}}\right)\left(1 + a\frac{(x-c)}{b}\right)^{-1-\frac{1}{a}}$$

Results- Inner Mongolia

Fixed α, varied π and β – scatter plots of $\{(x_i, f_i)\}$





- When π is small, limited stop loss or its variants are optimal, with the increase of π, the optimal form may become stop loss without a limit.
- With the increase of β, the optimal form may become limited stop loss from stop loss.
- Small π and large β play similar roles in the process of optimal reinsurance design.
- The reality is, fairly large β and small π (strict reinsurance premiums budget).

Robustness test- Inner Mongolia

- For $\{(x_i, f_i)\}$, fit $\mathrm{f(x)} = \min\{\mathrm{c(x-d)_+, m}\}$, we replicate the random samples 500 times independently to obtain 500 independent estimates of $\hat{c}$ and $\hat{d}$ using ε= 0.001.
- For one simulation, if $|\mathrm{f}(x_i) - f_i| \le \varepsilon$ for i=1,2,…,1000, one admission is obtained, admissibility=times of admission/500.

$\beta = 0.3$, samples size=1000, ε= 0.001					
α	π	admissibility	$\bar{c}$	$\bar{d}$	$\overline{d+m}$
5%	3.00%	100.00%	1.00	88.69%	107.81%
	3.75%	100.00%	1.00	85.03%	107.54%
	4.00%	100.00%	1.00	84.16%	107.93%
10%	3.00%	100.00%	1.00	81.67%	94.42%
	3.75%	100.00%	1.00	79.41%	94.94%
	4.00%	100.00%	1.00	78.80%	95.28%
20%	3.00%	100.00%	1.00	72.14%	80.42%
	3.75%	100.00%	1.00	70.11%	80.10%
	4.00%	100.00%	1.00	69.80%	80.49%

Results- Inner Mongolia

Optimal reinsurance arrangement of Inner Mongolia under different constraints							
$\boldsymbol{\beta}$		30%		40%		50%	
$\boldsymbol{\alpha}$	$\boldsymbol{\pi}$	d	$d+m$	d	$d+m$	d	$d+m$
5%	3.00%	89%	108%	91%	108%	93%	109%
	3.75%	85%	108%	88%	109%	90%	108%
	4.00%	84%	108%	87%	109%	90%	109%
10%	3.00%	82%	94%	83%	94%	84%	95%
	3.75%	79%	95%	81%	95%	83%	96%
	4.00%	79%	95%	80%	95%	82%	96%
20%	3.00%	72%	80%	73%	80%	73%	80%
	3.75%	70%	80%	71%	81%	72%	81%
	4.00%	70%	80%	71%	81%	72%	81%

Conclusion

- When the primary insurer's loss function, the reinsurance premium calculation principle, risk measure ρ are given, α, β, π all affect the optimal reinsurance design.
- When strict constraint on reinsurance premiums budget π are implemented (which is often the reality), Limited Stop Loss Reinsurance is optimal.
- Reinsurance premium calculation principle and safety loading coefficient of reinsurers play important roles in the optimal reinsurance decision-making process.

Thank you for your attention