Financial Engineering for the Farm Problem

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Incomplete draft: April 2013
IARFIC Conference Submission

Abstract

In this paper we provide a general discussion of how techniques in financial engineering can be used to investigate the economic costs of farm programs and to aid in the design of new financial products. In this paper we illustrate how financial engineering can be used to price USAs complex price stabilization formula and to illustrate its use in the design of new products. Dairy producers confront increasing price risks from both inputs and outputs as the prices of milk, corn and soybean become more volatile in recent years. These risks significantly affect dairy producer's profit margin. This paper examines the effectiveness of two new options products – milk-to-corn price ratio and milk-to-feed price ratio contract in protecting the profit margin for dairy producers with only one hedging position. We develop a simple farm profit model, and test the six scenarios of dairy farm profits. Finally, using a sample 36 New York State dairy farms from 1996 to 2010, we empirically evaluate the effect these instruments in managing price risk and protecting the farm margins in a mean and variance framework.

Keywords: Profit Hedging, Risk Management
JEL classifications:
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1. Introduction

Dairy producers confront various sources of risk. Among those, the uncertainty associated with the future cash price of a commodity is known as price risk. Dairy farm profits are not only affected by the price risk from the output milk price received, but also influenced by the volatility in input prices. Dairy feed, which mainly consists of corn and soybean, is one of the most important inputs for dairy producers. As a non-storable commodity, there could be large change in milk prices in reaction to changes in market fundamental. On the other hand, high costs and high volatility of feed prices are threatening the survival of dairy farm business.

While there have been ample papers written on various aspects of dairy policy, it is surprising that few papers recognize the broader aspect so financial engineering to resolve the farm problem. In this paper we show how Ito's Lemma can be used in direct and simulated form to price out not only the value of milk stabilization policies, but to develop new products to solve the farm problem.

The increase in the price volatility of milk, corn and soybean in recent years[1], poses an increasing threat to the business survival of dairy producers. As shown in Figure 1, 2 and 3, New York State all milk prices have not increased much for the past 15 years[2] while corn and soybean prices have soared since 2006. While the rising feed costs may be temporary, if dairy farmers in New York are inadequately hedged against this risk, shrinking profit margins may lead them to the brink of bankruptcy.

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[1] Corn, soybean and alfalfa hay prices are converted from dollar per bushel to dollar per hundred pounds in this thesis. According to U.S. commercial bushel sizes, corn has a standard of 56 pounds per bushel, soybean 60 pounds per bushel, alfalfa hay 2,000 pounds per ton.

[2] The mean and standard deviation for monthly New York State all milk prices from 1996 to 2010 is $15.29/cwt and $2.59/cwt respectively, for U.S. average corn price is $4.86/100 lbs and $1.65/100 lbs, for U.S. average soybean price is $11.51/100 lbs and $3.69/100 lbs.

In this paper, we propose two new options contracts, R1 and R2, for the explicit propose of of protecting profit margin for dairy producers with only one hedge position in market. R1 is the milk-corn contract, which is based on the price ratio between one Class III milk futures and one Corn futures contract. R2 is the milk-feed contract, which is based on the price ratio between one Class III milk futures and a combination of Corn futures contract and Soybean futures contract. The weights of Corn futures contract and Soybean futures contract sum up to one and are based on the feed ration of a dairy cow, which will be specified in details in Chapter 3.

This remainder of the paper is organized as follows. Section 2 motivates the development of a new option contract to manage profit risk facing dairy farmers, by presenting the background information on milk-feed ratio, and documents the increasing in price volatility. Section 3 presents a simple farm profit model, and develops the theoretical framework for constructing and implementing the options contracts to dairy operation. Section 4 describes the data and methods in the empirical analysis of the model. Section 5 shows the results. Section 6 concludes, draws additional implications, and discusses agendas for further research.

Section 2: Milk and Feed Price Volatility

Section 2.1 Milk-feed ratio

Milk-feed ratio is a common measure to assess the profitability of a dairy farm. According to USDA, the milk-feed ratio is the number of pounds of 16 percent protein-mixed dairy feed equal in value to 1 pound of whole milk. High value for this ratio indicates that feed is relatively cheap to milk and vice versa. The mixed dairy feed for the ratio consists of 51 pounds of corn, 8 pounds of soybeans and 41 pounds of alfalfa hay. The major feed components of corn and soybeans account for 83 to 91 percent of the total ingredients in the rations in terms of value[3]. Thus, the dynamics for the ratio of milk price to weighted average of corn and soybean price shows the variability in the profitability of dairy farmers. Figure #4 shows the historical dynamics for this profitability ratio.

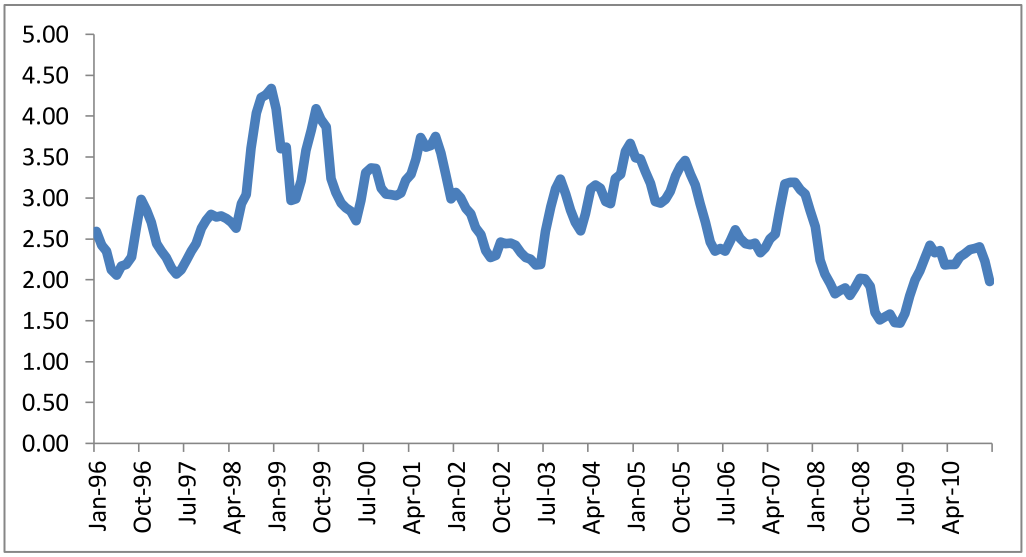

Figure 1 U.S. Average Milk-Feed Monthly Price Ratio from 1996 to 2010

Figure #4 shows <DOES IT REALLY SHOW THIS? – it's not that obvious> that milk-feed ratio has decreased in recent years. This is mainly the result of dramatic rise in feed prices. Low values of this ratio are signs that feed is relatively expensive to milk price. In this case, a relatively lower dairy income over feed costs should be expected. In figure 5, annual U.S. average milk-feed ratio is plotted against the average net farm income without appreciation for the same 79 New York dairy farms from 2000 to 2009. The trend generally shows that when milk-feed ratio is low during the year, net farm income is usually low. Therefore, we argue for the importance of a hedging instrument that allows dairy farmers to directly

[3] Source from "Tracking milk prices and feed costs" by Kenneth Bailey and Virginia Ishler, Pennsylvania State University

insure against this risk.

Section 3: An Alternative Profit Hedging Instrument for Dairy Farmers

The previous section showed the dramatic price volatility in the milk to feed ratio that dairy farmers faced over the last decade. The movement in this ratio directly impacts the profitability of dairy farmers. This section proposes two new futures contracts, one based on the milk-corn price ratio and the other based on the milk-feed (corn and soybean) price ratio, are introduced for dairy farm profit hedging purposes. Again, we argue that the relative value between milk and dairy feed is the key determinant of dairy farm profit from dairy operations. The feed considered in this paper are corn, soybean and alfalfa hay which are the main grain to feed cows, provide protein and fiber respectively. Corn and soybean futures are also among the most important agricultural commodities traded on CME in terms of trading volume and open interests[4]. Alfalfa hay is not a tradable on CME, thus its costs could not be hedged using corresponding futures. Therefore, only milk, corn and soybean are considered when choosing hedging instruments. The first futures contract R1 is based on milk-corn price ratio which serves the purposes of hedging milk production and a partial hedge of feed costs. It is shown that selling R1 is different from taking a short position in milk futures and a long position in corn futures at the same time later in this chapter. The second futures contract R2 is based on milk-feed price ratio which provides dairy farms an additional tool for hedging the relationship between the revenue from selling milk and costs from feeding cows.

Section 3.1: Hedging Dairy Profits with Milk-Feed Ratios

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[4] Daily trading volume of corn and soybean futures on the Chicago Board of Trade is much higher than the volume in milk contracts.

We first propose a direct hedging instrument based on the ratio of the price of milk to corn. We define this ratio, R1, to be:

$$R_1 = \frac{m}{c} \tag{1}$$

where m and c are the referencing futures price of Class III milk and corn. The idea of ratio is intuitive. By taking a short position in R1, dairy farmers are protecting themselves from potential decrease in the milk to corn price ratio. Contract R1 would help dairy producer hedge against price risk of milk and part of the feed (corn).

However, dairy farmers are not always bound to using 100% corn as feed. And in fact, feed can be any combination of <XXXXXX>. Therefore, we develop a more complicated ratio, R2, to proxy for the dairy profitability. While this model is easily extendable to include the cost of many different feeds, <in the interest of parsimony>, we use a combination of corn and soybean feed costs. R2 is defined as,

$$R_2 = \frac{m}{\alpha c + (1-\alpha)s} \tag{2}$$

where m, c, s are the referencing futures price of Class III milk, corn and soybean and α is the percentage of corn relative to soybean used in the feed.

Section 3.2: Price Dynamics for Milk, Corn, and Soybean

To model the price behaviors of R1 and R2, we begin with a model of the price dynamics for the underlying assets of interests, namely milk, corn, and soybean:

$$\frac{dm_t}{m_t} = \mu_m dt + \sigma_m dz_m \tag{3}$$

$$\frac{dc_t}{c_t} = \mu_c dt + \sigma_c dz_c \tag{4}$$

$$\frac{ds_t}{s_t} = \mu_s dt + \sigma_s dz_s \tag{5}$$

where m, c, s represents milk, corn and soybean respectively;

$\frac{dm_t}{m_t}$ = percentage change in milk futures price during time dt;

μ_m= expected growth rate in Class III milk futures price (drift term);

σ_m= variance of percentage return;

z_m is a Wiener process;

To develop the stochastic process of R1 and R2 given the price dynamics of milk, corn, and soybeans specified in Equations (7), (8), and (9), we exploit Ito's Lemma. Apply Ito's Lemma, it can be easily shown (see Appendix 1) that the process followed by $R_1 = \frac{m}{c}$ is derived as,

$$\frac{dR_{1,t}}{R_{1,t}} = (\mu_m - \mu_c - \sigma_m\sigma_c\rho_{mc})dt + \sigma_m dz_m - \sigma_c dz_c \tag{6}$$

where ρ_{mc} is the coefficient of correlation between the two processes of milk and corn futures prices. Hence, over any finite time interval T, the percentage chan

ge in R_1 is normally distributed with mean $(\mu_m - \mu_c - \sigma_m\sigma_c\rho_{mc})T$ and variance $(\sigma_m^2 + \sigma_c^2 - 2\sigma_m\sigma_c\rho_{mc})T$. R_1 follows Ito's process .

$$\widetilde{R_{1,t}} = \widetilde{R_{1,t-1}}\, exp((\mu_m - \mu_c - \sigma_m\sigma_c\rho_{mc})\, dt + N(0,1)\sigma_m\sqrt{dt} - N(0,1)\sigma_c\sqrt{dt}) \tag{7}$$

It is worth noticing that by taking a short position in milk futures and a long position in corn futures, dairy farmers are also protecting themselves from decreasing milk prices and increasing feed costs. It could be represented by creating a synthetic futures contract $\bar{R}$, where $\bar{R} = \alpha m - \beta c$. The Ito's processes generated from $\bar{R}$ are different from R_1.

$$d\overline{R_t} = (\alpha\mu_m m_t - \beta\mu_c c_t)dt + \alpha\sigma_m m_t dz_m - \beta\sigma_c c_t dz_c \tag{8}$$

Compared to taking a short position in milk futures and a long position in corn futures at the same

time, R1 reduces transaction costs as farmers only need to adjust the position of R1 should the prices of milk or corn change. <Need to say more> Similar to the derivation for R_1 based on milk-corn price ratio, apply Ito's lemma, the Ito's process followed by R_2 is written as,

$$\widetilde{R_{2,t}} = \widetilde{R_{2,t-1}}\, exp\Bigg((\mu_m - \frac{\alpha c_t}{\alpha c_t + (1-\alpha)s_t}(\mu_c + \sigma_m\sigma_c\rho_{mc}) - \frac{(1-\alpha)s_t}{\alpha c_t + (1-\alpha)s_t}(\mu_s + \sigma_m\sigma_s\rho_{ms}) + \frac{2\alpha(1-\alpha)c_t s_t}{(\alpha c_t + (1-\alpha)s_t)^2}\sigma_c\sigma_s\rho_{cs})dt + N(0,1)\sigma_m\sqrt{dt} - N(0,1)\frac{\alpha c_t}{\alpha c_t + (1-\alpha)s_t}\sigma_c\sqrt{dt} - \frac{(1-\alpha)s_t}{\alpha c_t + (1-\alpha)s_t}\sigma_s\sqrt{dt}\Bigg) \quad (9)$$

Where,

$$c_t = c_{t-1}\, exp\left((\mu_c - \frac{1}{2}\sigma_c^2)dt + N(0,1)\sigma_c\sqrt{dt} \right)$$
$$s_t = s_{t-1}\, exp\left((\mu_s - \frac{1}{2}\sigma_s^2)dt + N(0,1)\sigma_s\sqrt{dt} \right)$$

Section 4: Pricing Futures and Options on Milk-Feed Ratio

Section 4.1: Futures Contract

In this section, we specify the price behavior of commodities that will be important for the pricing of our proposed hedging instrument. Little common agreement has been reached on the data generating process of futures price, however, Tomek and Peterson's survey of risk management in agricultural markets provides an excellent review of commodity price models. In their review, the simplest model states that the price of a futures contract at current time t is the expected value of spot price at contract maturity T conditional on the information available at current time.

$$F(t,T) = E[P(T)|X(t)] \quad (10)$$

where $F(t,T)$ = price of a futures contract at time t that expires at time T;

$X(t)$ = information available at time t;

$E[P(T)]$ = expected value of spot price at contract maturity T;

This equation implies that the futures price is an unbiased estimate of terminal price.

Another model proposes that there exists non-zero risk premium in futures price (Tomek). The risk premium could vary through time.

$$E[F(T)|\Phi(t)] = F(t,T) + \gamma(t,T) \quad (11)$$

where $\gamma(t,T)$ = risk premium at time t. However, risk premium may be so small that it could not be found statistically in a futures market. For the purposes of this work, the arguments may not be quite essential as the proposed futures contracts are defined as the ratios of existing futures prices. However, the assumption of futures price behavior is crucial if pricing options on the proposed futures contracts, where the futures price is assumed to follow a random walk.

We assume that futures prices to be unbiased estimates. The basic hypothesis of the random walk theory is that price changes occur randomly and are not correlated from each other. The applicability of a random walk is based on the assumption that the futures market fits in the concept of an efficient and arbitrage-free market, that the futures price at a given time reflects all the information available at that time. Time-series of price changes have zero auto-correlations. New information that affects the futures price happens randomly and cannot be predicted in advance. In futures market, the random walk can be expressed as,

$$F_t - F_{t-1} = \varepsilon_t \quad (12)$$

where F_t is the discrete futures price series, ε_t has a mean of zero and is uncorrelated with ε_{t+k} $(k \neq 0)$. This model simplifies the actual data generating process for futures prices by assuming that the price series behaves as a simple stochastic process.

To describe the random walk in futures price series, geometric Brownian motion with constant drift and volatility is often applied as a standard approach to model time series of financial instruments.

$$\frac{dF}{F} = \mu dt + \sigma dz \qquad (13)$$

where μ is the expected growth rate in the futures price F, σ is its volatility and dz is a Wiener process. This is the underlying assumption when pricing options on futures using the Black-Scholes model. In this work, futures prices are assumed to be unbiased estimates. Thus, the mean of the percentage price change is zero. An implication of unbiasedness from the above equation is that the drift term μ is zero. Turvey (2006) investigates the existence of a geometric Brownian motion in 17 agricultural commodity price time series. The results indicate that the null hypothesis of ordinary Brownian motion cannot be rejected for 14 of 17 series.

Section 4.2: Options Contract

<WHAT do we want to do here?>

Section 5: Farm Profit Model and Sources of Risk

Net farm income is a common practice to measure the financial year result of dairy farm's whole operations. In United States agricultural policy, net farm income refers to the return (both monetary and non-monetary) to farm operators for their labor, management and capital, after all production expenses have been paid[5]. Since our hedging focuses on price risk reduction, farm profit from dairy operations is derived rather than using net farm income. The farm profit from dairy operations without hedging is defined as,

$$\pi = milk\ sales - feed\ costs - other\ operating\ costs \qquad (14)$$

The total feed costs have three components: costs of corn, costs of soybean and costs of alfalfa hay. Feed

[5] Source: http://en.wikipedia.org/wiki/Net_farm_income

costs for farm i can be written as,

$$C_i = Q_{ci}P_{ci} + Q_{si}P_{si} + Q_{hi}P_{hi} \quad (15)$$

where P_{ci} = purchase price of corn;

P_{si} = purchase price of soybean;

P_{hi} = purchase price of alfalfa hay;

Assume all dairy producers feed cows based on a standard dairy ration with fixed quantity of corn, soybean and alfalfa hay, then assume $Q_c = aQ_m$, $Q_s = bQ_m$, $Q_h = cQ_m$ for all dairy farms, where Q_m is the quantity of milk produced and sold during a time period of T, Q_c, Q_s and Q_h are the quantity of corn, soybean and alfalfa hay purchased during the corresponding time period. Also assume the feed purchased are consumed entirely during this time period with no inventory.

Substitute $Q_c = aQ_m$, $Q_s = bQ_m$, $Q_h = cQ_m$ into (16),

$$C_i = aQ_{mi}P_{ci} + bQ_{mi}P_{si} + cQ_{mi}P_{hi} \quad (16)$$

Suppose other operating costs K is a fixed proportion of the quantity of milk sold, $K = kQ_m$, k is a constant. The farm profit from dairy operations can be written as,

$$\pi_i = Q_{mi}P_{mi} - (Q_{ci}P_{ci} + Q_{si}P_{si} + Q_{hi}P_{hi}) - K_i \quad (17)$$

Thus,

$$\pi_i = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) \quad (18)$$

In other words, the equation for farm profits could be established as,

$$\begin{aligned} Farm\ profit\ (\$/cwt) &= milk\ volume\ (cwt) \times (milk\ price(\$/cwt) \\ &- feed\ costs(\$/cwt) \\ &- other\ operating\ costs\ (\$/cwt)) \end{aligned} \quad (19)$$

The variance of farm profit model described above is given by:

$$\sigma_{\pi_i}^2 = \sigma_{Q_{mi}}^2(\sigma_{P_{mi}}^2 + a^2\sigma_{P_{ci}}^2 + b^2\sigma_{P_{si}}^2 + c^2\sigma_{P_{hi}}^2 + k^2 - 2acov(P_{mi}, P_{ci}) - 2bcov(P_{mi}, P_{si}) - 2ccov(P_{mi}, P_{hi}) + 2abcov(P_{ci}, P_{si}) + 2bccov(P_{si}, P_{hi}) + 2accov(P_{ci}, P_{hi}) - 2cov(P_{mi}, k_i) + 2acov(P_{ci}, k_i) + 2bcov(P_{si}, k_i) + 2ccov(P_{hi}, k_i)) \tag{20}$$

The derivation indicates that the sources of risk for individual dairy farm profit include production risk, price risk and risk from operating costs. Production risk comes from the uncertainty of the quantity of milk produced and sold which could be different from the expectation of the dairy producer. However, the production risk for dairy producers is relatively small compare to other agricultural products producers as it is not significantly influenced by unpredictable factors, such as weather. Price risk comes from the volatility of milk price and prices for each of the feed component. It could be fairly significant as the prices of agricultural commodities tend to be volatile and difficult to be forecasted accurately. Operating costs, which mainly characterize labor and machinery costs, could vary from time to time based on the economic condition and farm operating efficiencies.

The market hedging instruments discussed in this thesis, namely futures contracts, could only be used to hedge against the price risk faced by dairy producers, leaving the other sources of risk unhedged. In addition, futures contract may not perfectly hedge against all the price risks because of the existence of basis risk. Therefore, it is reasonable to assume the quantity of milk produced and operating costs are independent from the hedging decision. Production risk is ignored in the derivation of optimal hedge ratio. Risk from operating costs is isolated by assuming operating costs are proportional to the quantity of milk produced and the ratio remains the same across years for the same farm. These assumptions serve to better reflect the objective of hedging price risk with futures contract.

Section 5.1: Optimal Hedge Ratio

The objective of this paper is to compare the effectiveness of hedging using R1 or R2 futures with the conventional hedging using milk, corn or soybean futures. Consider a dairy farm that expects the quantity of milk production over the next period to be $Q_{m,t-1}$ at time $t-1$ and takes out futures positions in a futures contract at the same time. $P_{m,t}$ is the spot price of milk in period t. $F_{m,t}$ is the futures price in period t for delivery at some future date. The futures positions are liquidated at time t.

The dairy farm profit in period t with hedging position in one futures contract can be denoted as,

$$\pi_t = Q_{m,t-1}P_{m,t} - C - (F_{m,t} - F_{m,t-1})h_{t-1}Q_{m,t-1} \quad (21)$$

where h_{t-1} is the hedge ratio to the quantity of milk production $Q_{m,t-1}$. If $h > 0$, it implies a short position. If $h < 0$, it implies a long position. C is a cost function of feed costs and operating costs.

The objective is to maximize a linear function of the mean and variance of the farm profit in the next period by choosing the hedge ratio at the beginning of this period, conditional on available information,

$$\max_{h_{t-1}} E(\pi_t|X_{t-1}) - \frac{\lambda}{2} var(\pi_t|X_{t-1}) \quad (22)$$

where X_{t-1} is a set of information available at time $t-1$, λ is a measure of the dairy farm's risk aversion. According to Myers and Thompson, if

(1) C is an increasing and convex cost function;

(2) Quantity of milk production is independent from hedging decision;

(3) Futures market is unbiased, $E(F_t|X_{t-1}) = F_{t-1}$;

then the optimal hedge ratio equals to,

$$h = \frac{\sigma_{P_m F_m}}{\sigma_{F_m}^2} \quad (23)$$

If the futures market is biased, the derived hedge ratio satisfies the minimum variance of the profit function but is not mean-variance efficient (Heifer).

Six scenarios of the dairy farm profit are being compared in this paper. Five scenarios use futures contracts as hedging instrument. The six scenarios are:

(1) Hedge milk sales only: short class III milk futures

(2) Hedge costs of corn only: long corn futures

(3) Hedge milk sales and costs of corn simultaneously: short class III milk futures and long corn futures

(4) Hedge milk sales and costs of corn simultaneously: short futures contract R1

(5) Hedge milk sales and feed costs simultaneously: short futures contract R2

(6) No hedging

In the farm profit model of this paper, the cost function does not follow the properties as an increasing and convex function. As a result, the covariance between the variable in the cost function and the milk spot and futures price will influence the hedge ratio. The derivation of the optimal hedge ratio should be conducted for each hedging strategy rather than applying the simple hedge ratio.

(1) Hedge milk sales only: short class III milk futures

The profit function for dairy farm i is,

$$\pi_i^{h1} = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) - (F_{m,t} - F_{m,t-1})h_1 Q_{mi} \quad (24)$$

Derive the variance of π_i^{h1} as,

$$\sigma^2_{\pi_i^{h1}} = Q_{mi}^2\sigma_{P_{mi}}^2 + a^2Q_{mi}^2\sigma_{P_{ci}}^2 + b^2Q_{mi}^2\sigma_{P_{si}}^2 + c^2Q_{mi}^2\sigma_{P_{hi}}^2 + h_1^2Q_{mi}^2\sigma_{F_m}^2 - 2aQ_{mi}^2cov(P_{mi},P_{ci}) - 2bQ_{mi}^2cov(P_{mi},P_{si}) - 2cQ_{mi}^2cov(P_{mi},P_{hi}) + 2abQ_{mi}^2cov(P_{ci},P_{si}) + 2bcQ_{mi}^2cov(P_{si},P_{hi}) + 2acQ_{mi}^2cov(P_{ci},P_{hi}) - 2h_1Q_{mi}^2cov(P_{mi},F_{m,t}) + 2h_1aQ_{mi}^2cov(P_{ci},F_{m,t}) + 2h_1bQ_{mi}^2cov(P_{si},F_{m,t}) + 2h_1cQ_{mi}^2cov(P_{hi},F_{m,t}) \quad (25)$$

Note that Q_{mi} and k_i are independent from the hedging, so both are removed from the hedge ratio calculations.

Obtain the optimal h_1 by minimizing the variance of π_i^{h1}. Take the first derivative of $\sigma^2_{\pi_i^{h1}}$ with respect to h_1 and set the equation equal to zero.

$$\frac{\partial\sigma^2_{\pi_i^{h1}}}{\partial h_1} = 2h_1Q_{mi}^2\sigma_{F_m}^2 - 2Q_{mi}^2cov(P_{mi},F_{m,t}) + 2aQ_{mi}^2cov(P_{ci},F_{m,t}) + 2bQ_{mi}^2cov(P_{si},F_{m,t}) + 2cQ_{mi}^2cov(P_{hi},F_{m,t}) = 0 \quad (26)$$

Thus,

$$h_1 = \frac{1}{\sigma_{F_m}^2}[cov(P_{mi},F_{m,t}) - acov(P_{ci},F_{m,t}) - bcov(P_{si},F_{m,t}) - ccov(P_{hi},F_{m,t})] \quad (27)$$

Since the objective is to hedge milk sales by taking a short position in class III milk futures, it would be more reasonable to assume milk, corn, soybean and alfalfa hay price series are independent from each other when calculating the hedge ratio, i.e. the covariance between cash price of corn and futures price of milk is equal to zero. However, simultaneously estimated, time-varying hedge ratios may achieve more hedging effectiveness for the soybean processing margin which had been examined in previous literature. Then, it would be important to consider the correlations among the cash and futures price of all the commodities involved. As summarized by Tomek and Peterson from an intensive literature review, time-varying covariance estimation is costly and often does not result in greater hedging effectiveness relative to unconditional hedge ratios. For the purpose of this paper, the farm profit model is set up to be hedging with only one futures instrument and the hedge ratio is not estimated as time-varying hedge ratios. The interaction between the cash and futures price of different commodities are not considered.

The optimal hedge ratio for dairy farm i is,

$$h_1 = \frac{cov(P_{mi}, F_{m,t})}{\sigma^2_{F_m}} \tag{28}$$

(2) Hedge costs of corn only: long corn futures

The profit function for dairy farm i is,

$$\pi_i^{h2} = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) - (F_{c,t} - F_{c,t-1})h_2 Q_{mi} \tag{29}$$

Derive the variance of π_i^{h1} as,

$$\begin{aligned}\sigma^2_{\pi_i^{h2}} = {} & Q^2_{mi}\sigma^2_{P_{mi}} + a^2Q^2_{mi}\sigma^2_{P_{ci}} + b^2Q^2_{mi}\sigma^2_{P_{si}} + c^2Q^2_{mi}\sigma^2_{P_{hi}} + h_2^2Q^2_{mi}\sigma^2_{F_c} \\ & - 2aQ^2_{mi}cov(P_{mi}, P_{ci}) - 2bQ^2_{mi}cov(P_{mi}, P_{si}) \\ & - 2cQ^2_{mi}cov(P_{mi}, P_{hi}) + 2abQ^2_{mi}cov(P_{ci}, P_{si}) \\ & + 2bcQ^2_{mi}cov(P_{si}, P_{hi}) + 2acQ^2_{mi}cov(P_{ci}, P_{hi}) \\ & - 2h_2Q^2_{mi}cov(P_{mi}, F_{c,t}) + 2h_2aQ^2_{mi}cov(P_{ci}, F_{c,t}) \\ & + 2h_2bQ^2_{mi}cov(P_{si}, F_{c,t}) + 2h_2cQ^2_{mi}cov(P_{hi}, F_{c,t})\end{aligned} \tag{30}$$

Take the first derivative of $\sigma^2_{\pi_i^{h2}}$ with respect to h_2 and set the equation equal to zero.

$$\begin{aligned}\frac{\partial\sigma^2_{\pi_i^{h2}}}{\partial h_2} = {} & 2h_2Q^2_{mi}\sigma^2_{F_c} - 2Q^2_{mi}cov(P_{mi}, F_{c,t}) + 2aQ^2_{mi}cov(P_{ci}, F_{c,t}) \\ & + 2bQ^2_{mi}cov(P_{si}, F_{c,t}) + 2cQ^2_{mi}cov(P_{hi}, F_{c,t}) = 0\end{aligned} \tag{31}$$

Thus,

$$\begin{aligned}h_2 = {} & \frac{1}{\sigma^2_{F_c}}[cov(P_{mi}, F_{c,t}) - acov(P_{ci}, F_{c,t}) - bcov(P_{si}, F_{c,t}) \\ & - ccov(P_{hi}, F_{c,t})]\end{aligned} \tag{32}$$

Based on the assumptions above, the futures price of corn is uncorrelated with the cash price of milk, soybean and alfalfa hay. If the dairy producer only hedges the costs of corn, the optimal hedge ratio for dairy farm i is,

$$h_2 = \frac{-acov(P_{ci}, F_{c,t})}{\sigma^2_{F_c}} \tag{33}$$

(3) Hedge milk sales and costs of corn simultaneously: short class III milk futures and long corn futures

The profit function for dairy farm i is,

$$\pi_i^{hm,hc} = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) - (F_{m,t} - F_{m,t-1})h_m Q_{mi} - \left(F_{c,t} - F_{c,t-1}\right)h_c Q_{mi} \quad (34)$$

where h_m and h_c are the hedge ratio for class III milk futures and corn futures respectively.

The variance of $\pi_i^{hm,hc}$ is derived as,

$$\begin{aligned}\sigma^2_{\pi_i^{hm,hc}} = {} & Q_{mi}^2\sigma_{P_{mi}}^2 + a^2Q_{mi}^2\sigma_{P_{ci}}^2 + b^2Q_{mi}^2\sigma_{P_{si}}^2 + c^2Q_{mi}^2\sigma_{P_{hi}}^2 + h_m^2Q_{mi}^2\sigma_{F_m}^2 \\ & + h_c^2Q_{mi}^2\sigma_{F_c}^2 - 2aQ_{mi}^2cov(P_{mi}, P_{ci}) - 2bQ_{mi}^2cov(P_{mi}, P_{si}) \\ & - 2cQ_{mi}^2cov(P_{mi}, P_{hi}) + 2abQ_{mi}^2cov(P_{ci}, P_{si}) \\ & + 2bcQ_{mi}^2cov(P_{si}, P_{hi}) + 2acQ_{mi}^2cov(P_{ci}, P_{hi}) \\ & - 2h_mQ_{mi}^2cov\left(P_{mi}, F_{m,t}\right) - 2h_cQ_{mi}^2cov\left(P_{mi}, F_{c,t}\right) \\ & + 2h_maQ_{mi}^2cov\left(P_{ci}, F_{m,t}\right) + 2h_mbQ_{mi}^2cov\left(P_{si}, F_{m,t}\right) \\ & + 2h_mcQ_{mi}^2cov\left(P_{hi}, F_{m,t}\right) + 2h_caQ_{mi}^2cov\left(P_{ci}, F_{c,t}\right) \\ & + 2h_cbQ_{mi}^2cov\left(P_{si}, F_{c,t}\right) + 2h_ccQ_{mi}^2cov\left(P_{hi}, F_{c,t}\right) \\ & + 2h_mh_cQ_{mi}^2cov(F_{m,t}, F_{c,t})\end{aligned} \quad (35)$$

To solve for the optimal hedge ratios simultaneously, take the first derivative of the variance of π_i^{h1} with respect to h_m and h_c respectively and set both equations equal to zero.

$$\begin{aligned}\frac{\partial\sigma^2_{\pi_i^{hm,hc}}}{\partial h_m} = {} & 2h_mQ_{mi}^2\sigma_{F_m}^2 - 2Q_{mi}^2cov\left(P_{mi}, F_{m,t}\right) + 2aQ_{mi}^2cov\left(P_{ci}, F_{m,t}\right) \\ & + 2bQ_{mi}^2cov\left(P_{si}, F_{m,t}\right) + 2cQ_{mi}^2cov\left(P_{hi}, F_{m,t}\right) \\ & + 2h_cQ_{mi}^2cov(F_{m,t}, F_{c,t}) = 0\end{aligned} \quad (36)$$

$$\begin{aligned}\frac{\partial\sigma^2_{\pi_i^{hm,hc}}}{\partial h_c} = {} & 2h_cQ_{mi}^2\sigma_{F_c}^2 - 2Q_{mi}^2cov\left(P_{mi}, F_{c,t}\right) + 2aQ_{mi}^2cov\left(P_{ci}, F_{c,t}\right) \\ & + 2bQ_{mi}^2cov\left(P_{si}, F_{c,t}\right) + 2cQ_{mi}^2cov\left(P_{hi}, F_{c,t}\right) \\ & + 2h_mQ_{mi}^2cov(F_{m,t}, F_{c,t}) = 0\end{aligned} \quad (37)$$

Based on the assumptions above, the futures price of one commodity is uncorrelated with the cash price of another commodity in the model. We get two equations,

$$h_m\sigma_{F_m}^2 - cov\left(P_{mi}, F_{m,t}\right) + h_c cov(F_{m,t}, F_{c,t}) = 0 \quad (38)$$

$$h_c\sigma_{F_c}^2 + acov\big(P_{ci}, F_{c,t}\big) + h_m cov(F_{m,t}, F_{c,t}) = 0 \quad (39)$$

Solve the two equations and get,

$$h_m = \frac{cov(P_m, F_m)\sigma_{F_c}^2 + acov(P_c, F_c)cov(F_m, F_c)}{\sigma_{F_m}^2\sigma_{F_c}^2 - cov(F_m, F_c)^2} \quad (40)$$

$$h_c = \frac{cov(P_m, F_m)cov(F_m, F_c) + acov(P_c, F_c)\sigma_{F_m}^2}{cov(F_m, F_c)^2 - \sigma_{F_m}^2\sigma_{F_c}^2} \quad (41)$$

(4) Hedge milk sales and costs of corn simultaneously: short futures contract R1

The profit function for dairy farm i is,

$$\pi_i^{h3} = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) - (R_{1,t} - R_{1,t-1})h_3 Q_{mi} \quad (42)$$

Derive the variance of π_i^{h3}as,

$$\begin{aligned}\sigma_{\pi_i^{h3}}^2 = {} & Q_{mi}^2\sigma_{P_{mi}}^2 + a^2Q_{mi}^2\sigma_{P_{ci}}^2 + b^2Q_{mi}^2\sigma_{P_{si}}^2 + c^2Q_{mi}^2\sigma_{P_{hi}}^2 + h_3^2Q_{mi}^2\sigma_{R_1}^2 \\ & - 2aQ_{mi}^2cov(P_{mi}, P_{ci}) - 2bQ_{mi}^2cov(P_{mi}, P_{si}) \\ & - 2cQ_{mi}^2cov(P_{mi}, P_{hi}) + 2abQ_{mi}^2cov(P_{ci}, P_{si}) \\ & + 2bcQ_{mi}^2cov(P_{si}, P_{hi}) + 2acQ_{mi}^2cov(P_{ci}, P_{hi}) \\ & - 2h_3Q_{mi}^2cov\big(P_{mi}, R_{1,t}\big) + 2h_3aQ_{mi}^2cov\big(P_{ci}, R_{1,t}\big) \\ & + 2h_3bQ_{mi}^2cov\big(P_{si}, R_{1,t}\big) + 2h_3cQ_{mi}^2cov\big(P_{hi}, R_{1,t}\big)\end{aligned} \quad (43)$$

Take the first derivative of $\sigma_{\pi_i^{h3}}^2$ with respective to h_3 and set the equation equal to zero.

$$\begin{aligned}\frac{\partial\sigma_{\pi_i^{h3}}^2}{\partial h_3} = {} & 2h_3Q_{mi}^2\sigma_{R_1}^2 - 2Q_{mi}^2cov\big(P_{mi}, R_{1,t}\big) \\ & + 2aQ_{mi}^2cov\big(P_{ci}, R_{1,t}\big) + 2bQ_{mi}^2cov\big(P_{si}, R_{1,t}\big) \\ & + 2cQ_{mi}^2cov\big(P_{hi}, R_{1,t}\big) = 0\end{aligned} \quad (44)$$

Thus,

$$\begin{aligned}h_3 = \frac{1}{\sigma_{R_1}^2}[& cov\big(P_{mi}, R_{1,t}\big) - acov\big(P_{ci}, R_{1,t}\big) \\ & - bcov\big(P_{si}, R_{1,t}\big) - ccov\big(P_{hi}, R_{1,t}\big)]\end{aligned} \quad (45)$$

The definition of the price of R1 contract is the ratio of class III milk futures price divided by corn futures price. Cash and futures price are considered highly correlated for identical underlying commodity. The covariance between futures price of R1 contract and cash price of milk and corn should be considered as non-zero. Based on the assumptions above, the futures price of R_1 is uncorrelated with the cash price of soybean and alfalfa hay. If the dairy producer hedges the profit margin by taking a short position in futures contract R1, the optimal hedge ratio for dairy farm i is,

$$h_3 = \frac{cov\left(P_{mi}, R_{1,t}\right) - acov\left(P_{ci}, R_{1,t}\right)}{\sigma_{R_1}^2} \tag{46}$$

(5) Hedge milk sales and feed costs simultaneously: short futures contract R2

The profit function for dairy farm i is,

$$\pi_i^{h4} = Q_{mi}P_{mi} - Q_{mi}(aP_{ci} + bP_{si} + cP_{hi} + k_i) - (R_{2,t} - R_{2,t-1})h_4 Q_{mi} \tag{47}$$

Derive the variance of π_i^{h4}as,

$$\begin{aligned}\sigma_{\pi_i^{h4}}^2 = {} & Q_{mi}^2\sigma_{P_{mi}}^2 + a^2Q_{mi}^2\sigma_{P_{ci}}^2 + b^2Q_{mi}^2\sigma_{P_{si}}^2 + c^2Q_{mi}^2\sigma_{P_{hi}}^2 + h_4^2Q_{mi}^2\sigma_{R_2}^2 \\ & - 2aQ_{mi}^2cov(P_{mi}, P_{ci}) - 2bQ_{mi}^2cov(P_{mi}, P_{si}) \\ & - 2cQ_{mi}^2cov(P_{mi}, P_{hi}) + 2abQ_{mi}^2cov(P_{ci}, P_{si}) \\ & + 2bcQ_{mi}^2cov(P_{si}, P_{hi}) + 2acQ_{mi}^2cov(P_{ci}, P_{hi}) \\ & - 2h_4Q_{mi}^2cov\left(P_{mi}, R_{2,t}\right) + 2h_4aQ_{mi}^2cov\left(P_{ci}, R_{2,t}\right) \\ & + 2h_4bQ_{mi}^2cov\left(P_{si}, R_{2,t}\right) + 2h_4cQ_{mi}^2cov\left(P_{hi}, R_{2,t}\right)\end{aligned} \tag{48}$$

Take the first derivative of $\sigma_{\pi_i^{h4}}^2$ with respective to h_4 and set the equation equal to zero.

$$\begin{aligned}\frac{\partial\sigma_{\pi_i^{h4}}^2}{\partial h_4} = {} & 2h_4Q_{mi}^2\sigma_{R_2}^2 - 2Q_{mi}^2cov\left(P_{mi}, R_{2,t}\right) + 2aQ_{mi}^2cov\left(P_{ci}, R_{2,t}\right) \\ & + 2bQ_{mi}^2cov\left(P_{si}, R_{2,t}\right) + 2cQ_{mi}^2cov\left(P_{hi}, R_{2,t}\right) = 0\end{aligned} \tag{49}$$

$$\begin{aligned}h_4 = {} & \frac{1}{\sigma_{R_2}^2}\left[cov\left(P_{mi}, R_{2,t}\right) - acov\left(P_{ci}, R_{2,t}\right) - bcov\left(P_{si}, R_{2,t}\right)\right. \\ & \left. - ccov\left(P_{hi}, R_{2,t}\right)\right]\end{aligned} \tag{50}$$

The definition of the price of R2 contract is the ratio of class III milk futures price divided by a combination of corn futures price and soybean futures price. The price series of R2 contract is independent from alfalfa hay cash prices but correlated with milk, corn and soybean cash prices. If the dairy producer hedges the profit margin by taking a short position in futures contract R2, the optimal hedge ratio for dairy farm i is,

$$h_4 = \frac{cov(P_{mi}, R_{2,t}) - acov(P_{ci}, R_{2,t}) - bcov(P_{si}, R_{2,t})}{\sigma_{R_2}^2} \quad (51)$$

Options

A more ideal structure would be to write put options on the ratio.

Have shown that the cost squeeze is hedgeable using futures

Since we propose that the milk/corn ratio is a contract of its own marked-to-market to actual future prices and will thus be priced using Black's (not Black-Scholes) model and using